

On [A26]

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based on joint work by

AGIKMS (Atobe–Gan–Ichino–Kaletha–Minguez–S.)

<https://arxiv.org/abs/2410.13504>

Goal

We illustrate the idea of proof when

- $M = \mathrm{GL}_3(E) \times \mathrm{GL}_1(E) \subset P = MN_P \subset G = \mathrm{GL}_4(E)$,
- $\psi = \mathbf{1}_{L_E} \boxtimes S_3 \oplus \mathbf{1}_{L_E} \boxtimes S_1 \in \Psi(M)$, thus $\pi_\psi = \mathbf{1}_{\mathrm{GL}_3(E)} \boxtimes \mathbf{1}_{\mathrm{GL}_1(E)} \in \mathrm{Irr}(M)$,
- \times denotes parabolic induction; $I(a_1, \dots, a_4) := |\cdot|^{a_1} \times \cdots \times |\cdot|^{a_4}$,
- w_{12} is Weyl element, or the corresponding LIO, for cycle (12).

Goal: The following rectangle commutes.

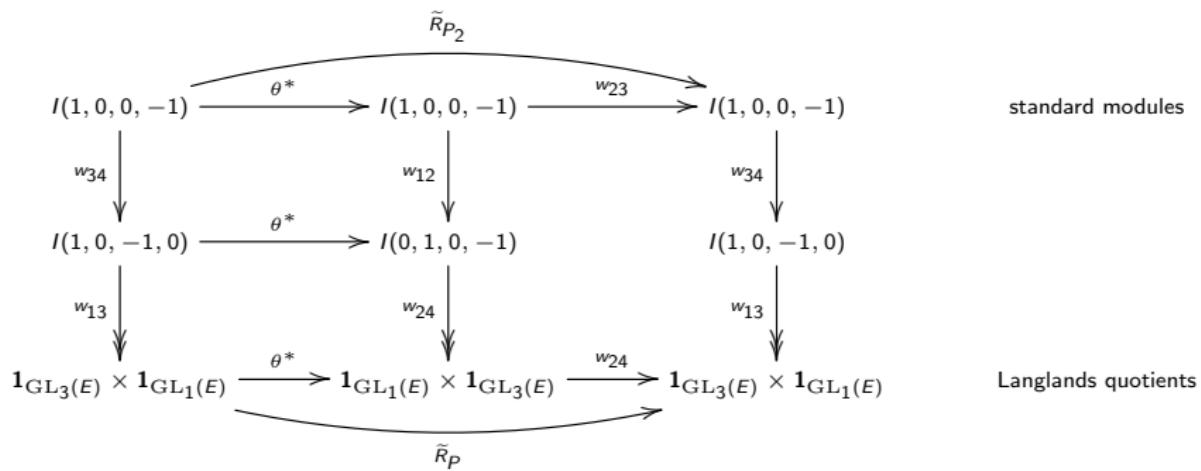
$$\begin{array}{ccc} I(1, 0, 0, -1) & \xrightarrow{\tilde{R}_{P_2}} & I(1, 0, 0, -1) \\ w_{34} \downarrow & & \downarrow w_{34} \\ I(1, 0, -1, 0) & & I(1, 0, -1, 0) \\ w_{13} \downarrow & & \downarrow w_{13} \\ \mathbf{1}_{\mathrm{GL}_3(E)} \times \mathbf{1}_{\mathrm{GL}_1(E)} & \xrightarrow{\tilde{R}_P} & \mathbf{1}_{\mathrm{GL}_3(E)} \times \mathbf{1}_{\mathrm{GL}_1(E)} \\ \curvearrowleft & & \curvearrowleft \\ I(-1, 0, 1, 0) & & I(-1, 0, 1, 0) \end{array}$$

standard modules
(understood)

Langlands quotients
(wanna understand)

Refine the diagram

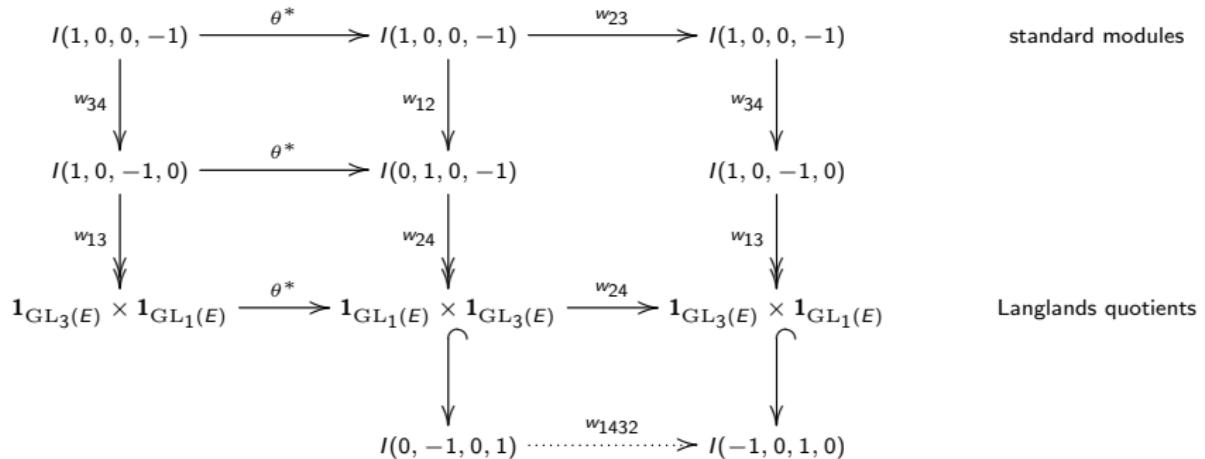
Decompose the twisted intertwiner \tilde{R} into θ^* and untwist.



Goal: The big rectangle commutes.

Left half isn't hard. Right half: why not obvious from multiplicativity?

Meromorphic continuation



- In meromorphic families, right half commutes by multiplicativity.
- At the point of interest (namely above), the bottom arrow is **not** defined.
- Solution: A careful 2-step specialization from 2-parameter memorphic families.