Periods in algebraic geometry and automorphic forms
Nisyros July 22nd-27th.

Minicourses

Henri Darmon and Luis García: Analytic cocycles and explicit class field theory.

Talk1 (Darmon) The first talk will set the stage by describing the two broad approaches to explicit class field theory growing out of (1) the theory of complex multiplication and its conjectural variants and (2) the Stark conjectures on leading terms of abelian L-series. We will focus on the archimedean approach of Bergeron, Charollois, and Garcia based on analytic cocycles which is one of the principal themes of this lecture series.

Talk 2 (García) In this talk I will describe a general topological approach to the construction of so-called Eisenstein cocycles. These cocycles provide a topological interpretation of certain special values of Hecke L-functions. Moreover, they reveal relations between the cohomology of arithmetic groups and arithmetic objects. I will briefly describe how the Mathai-Quillen approach to the Thom isomorphism underlies this construction. The talk is based on joint work with N. Bergeron and P. Charollois.

Talk 3 (García) In this talk I will describe our recent conjectures relating special values of the elliptic gamma function and explicit class field theory for cubic fields. Apart from discussing the conjecture and relevant numerical evidence, I will explain how it fits with the constructions discussed in the previous talk, and more broadly discuss the analogies with the ideas and methods developed by Henri and his collaborators. The talk is also based on joint work with N. Bergeron and P. Charollois.

Clément Dupont: Periods in Algebraic Geometry.

In this mini-course I will discuss various aspects of periods related to the cohomology of algebraic varieties. Special emphasis will be put on single-valued periods, which are relevant in the study of regulators and special values of L-functions. I will also report on recent joint work with Erik Panzer and Brent Pym on the (logarithmic) geometry of divergent period integrals.

Emmanuel Lecouturier and Romyar Sharifi: Eisenstein cocycles in motivic cohomology; Eisenstein cocycles for imaginary quadratic fields.

The work of Ribet on the converse to Herbrand's theorem gave a first indication of a subtle connection between the geometry of modular curves near the cusps and the arithmetic of cyclotomic fields, through the examination of the Galois representation attached to a newform congruent to an Eisenstein series. Our interest in these talks is a rather different connection, which runs in the opposite direction: an explicit map ϖ

taking modular symbols in the first homology of a modular curve to Steinberg symbols of cyclotomic units in a second K-group of a cyclotomic integer ring. The first speaker conjectured ϖ to be an isomorphism from the reduction of homology modulo an Eisenstein ideal. Fukaya and Kato proved much of the Eisenstein property by pulling back zeta maps to second étale cohomology groups of modular curves using a cusp.

The subject of our first talk is a different approach due to the first speaker and Venkatesh. We construct cocycles for $\mathrm{GL}_2(\mathbb{Z})$ valued in a limit of second motivic cohomology groups of opens in $\mathbb{G}_m^2$. The specialization of a restriction of this cocycle at a torsion point recovers ϖ and proves its Eisenstein property away from the level of the curve.

In the next two talks, we illustrate how the latter method can be extended and significantly refined to construct and prove the Eisenstein property of a long sought after map from the homology of a Bianchi space to a second K-group of a ray class field over an imaginary quadratic field F. We construct a collection of $\mathrm{GL}_2(\mathcal{O})$-cocycles, for $\mathcal{O}$ the ring of integers of F, valued in a limit of second motivic cohomology groups of opens in a product of two elliptic curves with CM by $\mathcal{O}$. This is joint work of the two speakers, Sheng-Chi Shih, and Jun Wang.

Research talks

Emmanuel Ullmo: Bi-$\overline{\mathbb{Q}}$-structure on Hermitian symmetric spaces and quadratic relations between CM periods.

We define a natural bi-$\overline{\mathbb{Q}}$-structure on the tangent space at a CM point on a Hermitian locally symmetric domain. We prove that this bi-$\overline{\mathbb{Q}}$-structure decomposes into the direct sum of 1-dimensional bi-$\overline{\mathbb{Q}}$-subspaces, and make this decomposition explicit for the moduli space of abelian varieties $\mathcal{A}_g$. We propose an Analytic Subspace Conjecture, which is the analogue of the Wüstholz's Analytic Subgroup Theorem in this context. We show that this conjecture, applied to $\mathcal{A}_g$, implies that all quadratic $\mathbb{Q}$-relations among the holomorphic periods of CM abelian varieties arise from elementary ones. We ilustrate the theory by the study of quadratic relations between CM periods of CM abelian varieties of generalized anti-Weyl type. As a consequence we prove that for any CM abelian variety A, there exists a CM abelian variety B such that the field generated by the periods of $A \times B$ is generated by relations of degree 2.

This is joint work with Z. Gao and A. Yafaev.

Masha Vlasenko: p-adic periods from naive viewpoint.

Expansion coefficients of differential form on an algebraic manifold over $\mathbb{Q}$ ”know” how many points there are over (almost) every finite field. We will demonstrate this principle at work for hypersurfaces and discuss phenomena of supercongruences and excellent Frobenius lifts. The talk is based on our joint work with Frits Beukers.

Kartik Prasanna: Motivic action for Siegel threefolds.

The goal of the talk will be to explain some recent joint work with Horawa on the relation between motivic cohomology and the coherent cohomology of Siegel threefolds. I will start by giving an introduction to the motivic action conjectures in two settings: (i) The cohomology of locally symmetric spaces (ii) The coherent cohomology of Shimura varieties There seems to be an interesting relation between these two settings, which I will discuss in the case of Bianchi and Siegel modular forms.

Giada Grossi: Euler systems, Iwasawa theory and periods.

I will give a general overview of the theory of Euler systems and explain how they are used to bound Selmer groups over the base field or over certain p-adic towers (e.g. cyclotomic, anticyclotomic over a quadratic imaginary field). The general conjecture/expectation is that their non-triviality can be detected by looking at certain values of L-functions (complex or p-adic). Depending on the audience's interests, I will then explain some new results either in the cyclotomic setting (for representations associated to Hilbert cuspforms, using motivic classes in the cohomology of Hilbert modular surfaces) or the anticyclotomic setting (for rational elliptic curves using special points on modular curves).

Raghuram: Betti–Whittaker periods.

I will begin by defining the Betti-Whittaker periods, which are certain periods attached to cohomological cuspidal representations of $GL(n)$ over a number field. I will then discuss variations on these periods. I hope to give a guided tour through rationality results on the special values of various automorphic L-functions where these periods play an important role. Intimately linked to these results on L-values are certain period relations. I will end the talk by presenting a recent result obtained in joint work with Giancarlo Castellano, generalizing a result of S.-Y. Chen, on the behaviour of the Betti-Whittaker periods under duality.

Tiago Fonseca: Motives for higher Green's functions.

Higher Green's functions are certain real-analytic functions in two variables on the upper half-plane which arise as resolvent kernels in the spectral theory of automorphic forms. Gross and Zagier observed and conjectured that these functions satisfy remarkable algebraicity properties at CM points; this is now a theorem by the works of Bruinier-Li-Yang, Li, and many others. In this talk, I'll explain how to view the values of higher Green's functions as periods of certain mixed modular motives. As an application, we give a motivic interpretation of Gross and Zagier's algebraicity conjecture, and hint on new identities suggested by Beilinson's conjectures. This is joint work in progress with Francis Brown.

David Loeffler: Spherical pairs, Iwasawa theory and p-adic families.

Many important constructions in Iwasawa theory - including Euler systems and p-adic L-functions - can be interpreted as cohomological incarnations of periods associated to spherical pairs of reductive groups. I will explain how this perspective can be used to unify a number of disparate existing results on variation of Euler systems and p-adic

L-functions in families of automorphic forms, and to prove many new results of the same type. This is joint work with R. Rockwood and S. Zerbes.

Xenia Dimitrakopoulou: Anticyclotomic p-adic L-functions for $U(n) \times U(n+1)$

I will explain how by p-adically interpolating the branching law for the spherical pair $(U(n), U(n) \times U(n+1))$, we can construct a p-adic L-function attached to Coleman families of cohomological automorphic representations of $U(n) \times U(n+1)$. Using the recent proof of the unitary Gan-Gross-Prasad conjecture, I will demonstrate that this p-adic L-function interpolates the square root of the central critical L-value, including anticyclotomic variation. Time allowing, I will explain how to extend this result to a Coleman family.

Yifeng Liu: Anticyclotomic Iwasawa main conjecture for Rankin-Selberg motives.

I will explain some recent results on the anticyclotomic Iwasawa main conjecture for Rankin-Selberg motives for $GL(n) \times GL(n+1)$. I will then discuss a main ingredient in the proof, namely, the generalization of Ribet's theorem on the level raising for modular curves at good primes to unitary Shimura varieties, and propose some questions.

James Arthur: Motives and automorphic representations.

We shall describe a conjectural concrete formula for the (pure) motivic Galois group. It would be a consequence of a parallel formula for an automorphic Galois group based on the principle of functoriality, that would classify automorphic representations. If time permits, we would add remarks on the possible extension of the motivic Galois group to a larger group that would include mixed motives, and possibly even exponential motives.