Trace Formulas
Nisyros 14-19 July, 2025
Abstracts

On [A25]
Hiraku Atobe (Kyoto University)

To establish local theorems in the endoscopic classification, Arthur uses a global argument. As a seed case, one needs to show local theorems for co-tempered parameters before the global argument. This is the topic in [A25], which is one of Arthur's papers in preparation. In this talk, I explain how to deduce two local theorems, endoscopic character relations and local intertwining relations, for co-tempered parameters from tempered parameters. This is a joint work with W. T. Gan, A. Ichino, T. Kaletha, A. Minguez and S. W. Shin.

The fine spectral expansion of the Rankin-Selberg period
Paul Boisseau (Aix-Marseille Université)

In 2011, Jacquet and Rallis proposed a strategy to prove the Bessel case of the global Gan–Gross–Prasad conjecture for unitary groups via a comparison of two relative trace formulae: one on unitary groups and the other on general linear groups. The latter involves the Rankin-Selberg period which provides integral representations of L-functions. In 2022, Beuzart-Plessis, Chaudouard and Zydor successfully implemented this approach, proving the conjecture for generic cuspidal representations. A key step in their argument is the computation of the contributions associated to these representations in the spectral expansions of both formulae.

In this talk, we will report on an ongoing work to derive the full spectral expansion of the Rankin-Selberg period on general linear groups. Our formula involves regularized periods of residual Eisenstein series à la Jacquet-Lapid-Rogawski and has applications to the non-generic version of the Gan-Gross-Prasad conjecture. The proof combines a truncation process introduced by Ichino and Yamana with contour integration techniques.

Counting Hitchin bundles for the group $\mathrm{SL}(n)$
Pierre-Henri Chaudouard (IMJ-PRG)

Hitchin bundles for the group $\mathrm{GL}(n)$ are pairs consisting of a vector bundle on a fixed projective curve and a twisted endomorphism. When the base field is finite, their counting is known; it is also closely related to the trace formula for Lie algebras. In the talk, I will focus on the problem of counting Hitchin bundles for the group $\mathrm{SL}(n)$. We will see that endoscopy for automorphic induction plays a role in obtaining a nice formula.

Automorphic Representations and Quantum Logic Gates
Rahul Dalal (Vienna)

Any construction of a quantum computer would require finding good sets of quantum logic gates: finite sets of $2^n-$by-2^n unitary matrices that efficiently and computably approximate arbitrary unitary matrices through short products. We explain a surprising connection between the these gate sets and automorphic representations. Using this, we explain how to input analytic bounds proven using the endoscopic classification to produce the first known constructions of optimal "golden" gate sets for more than one qubit.

This is joint work with Shai Evra and Ori Parzanchevski

Gan-Gross-Prasad cycles and p-adic relative-trace formulas
Daniel Disegni (Aix-Marseille Université)

Certain Rankin-Selberg motives of rank $n(n+1)$ are endowed with algebraic cycles arising from maps of unitary Shimura varieties. Gan-Gross-Prasad conjectured that these cycles are analogous to Heegner points, in the sense that their nontriviality should be detected by derivatives of L-functions. I will discuss another nontriviality criterion, based on p-adic L-functions. Under some local conditions, this variant can be established in a refined quantitative form, via the construction and comparison of two relative-trace formulas in p-adic coefficients. (Joint work with Wei Zhang.)

Beyond Endoscopy via Poisson Summation for GL(2) over a totally real number field
Melissa Emory (Oklahoma State)

Langlands proposed a strategy called Beyond Endoscopy to prove the principle of functoriality, which is one of the central questions of present-day mathematics. Langlands strategy of beyond endoscopy is a two-step process where the first step isolates the packets of cuspidal automorphic representations whose L-functions (for a representation of the dual group) have a pole at $s = 1$. The second step compares this data for two different groups and aims to determine functorial transfers. This talk deals with the first step. Altug worked with the group GL(2) over the rationals. This project generalizes Altug's result to GL(2) over a totally real number field (with some restrictions which we will discuss). In this talk we will emphasize some interesting differences between our work and Altug's work. This work is joint with Malors Espinosa-Lara, Debanjana Kundu, and Tian An Wong.

A new relative trace formula for a Shimura correspondence
Sol Friedberg (Boston College)

In this talk I discuss work towards establishing a new relative trace formula that is motivated by a conjectural Shimura correspondence from the triple cover of SL_3 to PGL_3. According to a 2001 conjecture of Bump, Friedberg and Ginzburg, the image of this lift should be detected by a "period" that, surprisingly, involves the automorphic minimal representation on split SO_8. As a first step towards establishing this, Offen and I gave the global functionals to be compared and explained how to reduce to a comparison of local orbital integrals, and we established the comparison for the unit elements in the respective spherical Hecke algebras. In our new work we have now established the Fundamental Lemma for Hecke correspondences.

Equidistribution of root numbers
Mathilde Gerbelli-Gauthier (Toronto)

In joint work with Rahul Dalal, we prove an equidistribution result for root numbers of (conjugate-)self-dual automorphic representations of GLN as the regular integral infinitesimal character grows. This is done in the framework of endoscopy and Arthur's trace formula, and the key trick is to generalize the observation that the root number is the eigenvalue of the Atkin-Lehner operator on the corresponding newform.

The local twisted Gan-Gross-Prasad conjecture
Hoang Le Nhat (Singapore)

The Gan-Gross-Prasad (GGP) conjecture studies a family of restriction problems for classical groups and proposes precise answers to these problems using the local and global Langlands correspondences. It also has a twisted variant in the equal-rank Fourier-Jacobi case, which is called the twisted Gan-Gross-Prasad conjecture. In this talk, motivated by the works of J.-L. Waldspurger and R. Beuzart-Plessis in Bessel models, I will introduce a local trace formula approach and how to use it to prove the twisted Gan-Gross-Prasad conjecture for tempered representations over nonarchimedean fields.

Endoscopy for symmetric varieties (minicourse)
Spencer Leslie (Boston College) & Wei Zhang (MIT)

In these talks, we discuss work toward a theory of endoscopy for certain spherical varieties. More precisely, we are interested in developing a notion of a "stable" relative trace formula for the purpose of affecting certain comparisons in the relative Langlands program. We will discuss some motivation for this, then sketch the pre-stabilization of the regular elliptic part of relative trace formulas associated to certain symmetric varieties. We will then formulate a definition of relative endoscopic datum, and formulate the natural local conjectures (smooth transfer and fundamental lemmata). We conclude by giving evidence for this theory in the case of unitary Friedberg–Jacquet periods, highlighting our recent joint work with Jingwei Xiao comparing the stable part of these RTFs to an RTF encoding central L-values of standard L-functions.

A Relative Trace Formula Approach to the Stable Trace Formula on the Unitary Group
Weixiao Lu (MIT)

Motivated by the study of the geometric side of the Jacquet–Rallis relative trace formula, we propose a relative trace formula on the general linear group that can be compared with the stable trace formula on the unitary group. In this talk, I will present this motivation as well as a joint work with Ryan Chen and Wei Zhang on its application to the diagonal cycle on the unitary Shimura variety.

BZSV duality and relative trace formula
Zhengyu Mao (Rutgers-Newark)

The relative trace formula is a powerful tool for both establishing Langlands functoriality and exploring the relationship between period integrals and L-values. The duality conjecture of Ben-Zvi, Sakellaridis, and Venkatesh (BZSV) offers a strong framework for the theory of the relative trace formula. We discuss work with S. Rallis and with C. Wan and L. Zhang on BZSV conjecture, which provides supporting evidence for the BZSV conjecture and suggests many new comparisons of relative trace formulas.

On A26
Sug Woo Shin (Berkeley)

In Arthur's endoscopic classification for quasi-split classical groups, a codename A26 is given to the local intertwining relation for the twisted group arising from GL(N) with respect to an involution. I will explain the context, the statement, and some ideas of proof. This talk is based on joint work with Hiraku Atobe, Wee Teck Gan, Atsushi Ichino, Alberto Minguez, and Tasho Kaletha.

Stable harmonic analysis and stable transfer
Matthew Sunohara (John Hopkins)

Stable transfer is the notion of transfer relevant for the beyond endoscopic comparisons of trace formulae envisioned by Langlands. I will present a stable Paley-Wiener Theorem for Harish-Chandra Schwartz functions and its application in demonstrating the existence of stable transfer operators. Understanding the structure and properties of these operators remains a central challenge, and I will discuss some perspectives on this problem.

Global twisted Gan-Gross-Prasad conjecture
Danielle Wang (Berkeley)

Let E/F and K/F be quadratic extensions of number fields, and let V be a skew-Hermitian space over E. The twisted Gan-Gross-Prasad conjecture considers the restriction of representations from $U(V_K)$ to $U(V)$. We discuss the relative trace formula approach to the global twisted GGP conjecture, proving it in the unramified case under some local assumptions. This is joint work with Weixiao Lu and Zhiyu Zhang.

Faltings heights and the sub-leading terms of adjoint L-functions
Wei Zhang (MIT)

The Kronecker limit formula is an equality relating the Faltings height of an CM elliptic curve to the sub-leading term (at s=0) of the Dirichlet L-function of an imaginary quadratic character. Colmez conjectured a generalization relating the Faltings height of any CM abelian variety to the subleading terms of certain Artin L-functions. In this talk we will formulate a "non-Artinian" generalization of (averaged) Colmez conjecture, relating the following two quantities: (1) the Faltings height of certain arithmetic Chow cycles on unitary Shimura varieties, and (2) the sub-leading term of the adjoint L-functions of (cohomological) automorphic representations of unitary groups U(n).

The $n = 1$ case of our conjecture recovers the averaged Colmez conjecture. We are able to prove our conjecture when $n = 2$ using a relative trace formula approach, which can be formulated for any n but we only understand the relevant archimedean local harmonic analysis when $n = 2$.

The "arithmetic relative Langlands" morally suggests that there should be a lot of other similar (at least conjectural) phenomena and we will mention some of them, including an ongoing work with Tony Feng and Zhiwei Yun on the (arithmetic) volume of Shimura varieties and moduli stacks of Drinfeld Shtukas.

Joint work with Ryan Chen and Weixiao Lu.